

MATHEMATICS TEST (variant A)

For each of the following tasks, choose the correct item (the only one item is correct) and its corresponding code (a, b, c, d, or e) write on the answer sheet.

Task 1. Simplify the expression

$$\frac{1 - z^3}{z^2 - 1}.$$

- a) $1 + z$ b) $-\frac{z^2 + 1}{z + 1}$ c) $1 - z$ d) $-\frac{z^2 + z + 1}{z + 1}$ e) $z - 1$

Task 2. Find a simple form of

$$\sqrt{\frac{c^3 \sqrt{d}}{d^3 \sqrt{c}}}.$$

- a) $c^4 \sqrt{d}$ b) $d^3 \sqrt{c^4}$ c) $c^{\frac{4}{3}} d^{-\frac{1}{4}}$ d) c e) $c^{\frac{7}{3}} d^{\frac{1}{4}}$

Task 3. Simplify the expression

$$\operatorname{tg} x + \operatorname{cotg} x - \frac{1}{\sin x \cos x}.$$

- a) $\operatorname{cotg} x$ b) $\frac{1}{\sin x} + \frac{1}{\cos x}$ c) $\operatorname{tg} x$ d) $\sin^2 x - \cos^2 x$ e) 0

Task 4. Let $i^2 = -1$. Find the distance between two complex numbers

$$1 + i \quad \text{and} \quad \frac{1}{i}.$$

- a) $\sqrt{2}$ b) $2\sqrt{2}$ c) 1 d) $\sqrt{5}$ e) $\sqrt{3}$

Task 5. What is the domain of the function

$$y = \frac{\cos x}{\log(1 + |1 - x^2|)} ?$$

- a) $(-\sqrt{2}, \sqrt{2})$ b) all reals c) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ d) $[-\sqrt{2}, \sqrt{2}]$ e) $(-1, 1)$

Task 6. What is the maximum value of function $y = 4x - x^2$?

- a) 2 b) 0 c) 4 d) no maximum e) $1/4$

Task 7. The function $y = x \sin x \cos x$ is:

- a) even b) odd c) periodic d) positive for all x e) negative for all x

Task 8. Choose the correct form of the function $f(x)$, if we know that $f(x)$ is defined only for positive x , takes all real values and is increasing on its domain:

- a) $f(x) = 1 + 3^x$ b) $f(x) = 1 + \log_3 x$ c) $f(x) = \left(\frac{1}{3}\right)^x$ d) $f(x) = \log_{\frac{1}{3}} x$ e) $f(x) = \sqrt{x}$

Task 9. How many different nine-digit numbers can be created by moving digits in the number 123454321?

- a) $5!$ b) $9!/16$ c) $9!$ d) $9!/8$ e) $4 \cdot 5!$

Task 10. There are 4 white and 3 black balls in the box. We randomly pick two balls. What is the probability that one is black and the other is white?

- a) $4/7$ b) $3/7$ c) $2/7$ d) $2/5$ e) $1/3$

Task 11. For which parameter $p \in \mathbb{R}$ has the quadratic inequality $x^2 + px + 9 \leq 0$ no solution in the real domain?

- a) $p \in (-\infty, -3) \cup (3, \infty)$ b) $p = (-9, 3)$ c) $p \in (-3, 9)$ d) $p \in (-6, 6)$ e) none of the previous answers is correct

Task 12. How many solutions are there for the equation $|x| = 3 - |x - 3|$ in domain \mathbb{R} ?

- a) 1 b) 2 c) 3 d) infinitely many solutions e) no solution

Task 13. How many solutions are there for the equation $\sin x \cos x = \frac{2}{3}$ in domain \mathbb{R} ?

- a) 1 b) 2 c) 4 d) infinitely many solutions e) no solution

Task 14. How many solutions are there for the equation $2^{2x^2} - 4^{x-1} = 0$ in domain \mathbb{R} ?

- a) 1 b) 2 c) 3 d) infinitely many solutions e) no solution

Task 15. What is the solution of the infinite equation

$$\sqrt{x} \cdot \sqrt[4]{x} \cdot \sqrt[8]{x} \cdot \sqrt[16]{x} \cdot \sqrt[32]{x} \cdot \dots = 16?$$

- a) $x = 2$ b) $x = 4$ c) $x = 8$ d) $x = 16$ e) no solution

Task 16. Let S_1 be a square with a side a . Let C be a circle inscribed into S_1 . Let S_2 be a square inscribed into C . Then the area ratio S_1/S_2 of the two squares is:

- a) $2\sqrt{2}$ b) 2 c) $\sqrt{2}$ d) $a/\sqrt{2}$ e) π/a

Task 17. What is the distance between points $A = [1, 6, 1]$ and $B = [5, 2, 3]$?

- a) 36 b) $\sqrt{104}$ c) 6 d) 2 e) $\sqrt{2}$

Task 18. Find all values of parameter p for which the vectors $u = (p, p + 1, 1)$ and $v = (p - 1, 1, -2)$ are perpendicular to each other.

- a) -1 b) 1 c) 2 d) $p = \pm 1$ e) $p = \pm 2$

Task 19. What is the slope of the line p which is given by points $A = [1, 0]$ and $B = [4, 5]$?

- a) $-5/3$ b) $-3/5$ c) $3/5$ d) $5/3$ e) 1

Task 20. What is the center C and radius r of a circle given by the equation $x^2 - 3x + y^2 + 4y = 0$?

- a) $C = [-3, 2], r = 5$ b) $C = [-\frac{3}{2}, 2], r = \frac{5}{2}$ c) $C = [3, -2], r = 5$ d) $C = [\frac{3}{2}, -2], r = \frac{5}{2}$ e) $C = [0, 0], r = 5$